THE LAWS OF RETURNS IN NEOCLASSICAL THEORIES OF GROWTH: A SRAFFIAN CRITIQUE[*]

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ABSTRACT

This paper critically examines exogenous and endogenous neoclassical theories of growth, paying particular attention to the assumptions about marginal returns to factors and returns to scale. It shows that the insistence on basing theories of growth on the traditional and flawed marginalist explanation of distribution extracts a heavy price, in the form of a number of deficiencies in the treatment of subjects like the relationship between accumulation and growth and increasing returns to scale, together with many implausible results and the need for artificial assumptions.

I. Introduction

Our purpose in the present paper is to present a critique, from a Sraffian point of view, of modern neoclassical theories of growth that have attracted so much attention over the last decade or so, both in their original exogenous version and in their endogenous version.[1]

We try to show how these growth theories pay a heavy price for being based on the neoclassical theory of distribution which is used, in their models, to ensure full

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[1] The literature on modern neoclassical theories of growth is vast. For our purposes here it is worth noting that the analytical structure of modern neoclassical growth models was clarified originally from a defensive point of view by Solow (1992) and in a Sraffian critical perspective by Cesaratto (1995, 1999a,1999b).

employment of the labour force and full utilization of capital equipment. The need to be consistent with such (in themselves quite unrealistic) full employment conditions forces these theories to rely on extreme and artificial assumptions about technology and technical progress.

We first outline (in section II) how the neoclassical theories of distribution and of the utilization of factors of production require decreasing marginal returns for capital accumulation (and in general for an increase in the use of any factor). Indeed, these decreasing marginal returns are the reason why many scholars are dissatisfied with exogenous growth models such as that of Solow, because they render the steady state rate of growth of the economy independent of the savings ratio.

Following this (in section III), we show how these theories retain decreasing returns for capital, in general, even when including externalities and economies of learning (as also embodied technical progress). Worse still, these variants of the Solow model that include externalities (and, more generally, any neoclassical model of growth with increasing returns of scale) inevitably generate the particularly implausible result of a strong positive causal relation between the rate of growth of population and the rate of growth of labour productivity, a relationship that seems to suggest that a permanent demographic explosion is the fastest route to catch up with the first world. We thus see that, in general, Solow's model is introduced with the hypothesis of constant returns to scale not because of analytical difficulties, but to avoid results that are even more implausible than those of the original model.

Finally (in section IV), we deal with the modern theories of endogenous growth, whose striking characteristic is the complete elimination of decreasing returns both to physical capital (AK models) and to the stock of 'knowledge' (Lucas), so that the factor that can be accumulated has constant marginal returns. We see that these models, which already depend on very artificial hypotheses about technology in which the labour force is assumed to be constant, require even more extreme and artificial hypotheses in the case of a growing labour force. These additional hypotheses must bring down to zero the marginal product of labour; else, with constant returns either for physical capital or 'knowledge', the economy will have ever-accelerating growth rates for any positive rate of growth of the labour force.

We close the paper (section V) with three additional critical remarks.

II. Decreasing Marginal Returns to Capital in the Neoclassical Theory of Exogenous Growth

II.1 Constant returns to scale and decreasing marginal returns for each factor

According to the neoclassical (or marginalist) vision of the operation of the market mechanism, in any competitive economy in which there is production, goods themselves are not scarce (because they can be produced) and therefore equilibrium prices necessarily have to cover the costs of production. In this context, the explanation of relative prices in terms of 'scarcity' requires that the factors of production that are used to produce the goods are what must be scarce. This notion of scarcity of 'factors' comes, as we know, from the idea that the endowments of the factors are exogenous and that it is possible to derive general equilibrium (excess) demand functions for these factors which are inversely related to their respective prices. This inverse relation between the relative price and relative quantity demanded of a factor, evidently, is based on the so-called principle of substitution through direct and/or indirect factor substitution.[2]

However, neoclassical theories of growth are usually based on models in which the economy produces a single homogenous product that is at the same time the only consumption and capital good in the economy, eliminating, at the very outset, an explicit analysis of indirect substitution and enabling the sole consideration of direct substitution. This is resorted to because the principle of direct substitution in production applies rigorously only in the case of methods of production that differ in the proportions of the factors used, whose quantities are specified in physical terms.

Even in such a context of severely limited generality, it is important to recall that the principle of substitution is derived from previous hypotheses about the exogenous nature of endowments of the factors of production and the existence of a multiplicity of available methods of production, all of them characterized by constant returns to scale

[2] Direct substitution occurs when a fall in the price of one factor induces the choice, for each good, of methods of production that use this factor more intensively. Indirect substitution happens when, even without changing the production methods, the fall in the relative prices of goods that use more intensively the factor that became relatively cheaper leads consumers to change their choices, in favour of consuming in larger quantity goods that use more intensively the factor that got cheaper (see Serrano, 2001).

(that is, methods in which the product would increase proportionally if the quantity of all factors are simultaneously expanded).

Decreasing marginal productivity, or decreasing returns to increases in each factor, keeping some other factor fixed, is not a hypothesis about technology but the combined result of the use of technology with constant returns to scale and exogenous endowment of factors.

As the quantity of the other factors is in principle exogenous, the utilization of additional quantities of a factor inevitably requires a change in the production method in use. This change will be in the direction of a method that has the disadvantage of having a lower product per unit of the factor that is varying, but at the same time uses proportionally less of the other factors, so that it becomes possible to increase production.

In fact, if it were always possible to automatically ensure parallel expansion of the quantities of all the other factors, the economy would continue using the same constant returns to scale methods on a larger scale. On the other hand, if several methods using different proportions of the factors were not available, the marginal product of an additional unit of a single factor would be zero, once full utilization is reached of the factor that is given.

Thus, if the endowment of a factor is exogenous and there is a multiplicity of methods (each one with constant returns to scale), we may deduce that the economy will produce with decreasing marginal products for each separate factor.

The demand of profit-maximizing producers for these factors, in such an economy, will be inversely related to the relative price of each factor. Because of decreasing marginal productivity, it will only be lucrative to increase the utilization of a factor, given the utilization of the others, if its price falls together with its marginal productivity.

A principle of substitution is thus derived, allowing for the construction of factor demand curves which, together with the factor endowments (and supply curves), simultaneously determine the relative price and quantity used of each factor.

In these theories, the prices of the factors tend to be proportional to their marginal productivities and there is a tendency towards full utilization of the endowments (including their owners' own demand).

For our purposes here it is important to stress that it is this kind of thinking about the determination of distribution, in terms of the supply and demand of factors, and particularly the hypothesis that all methods have constant returns to scale, that allows the neoclassical growth models to obtain full employment of all the factors (see Serrano, 2001, for a summary of the main objections to this presumed tendency). Therefore, constant returns to scale and decreasing returns for each factor are essential characteristics of the neoclassical explanation for the operation of the competitive market mechanism, and not hypotheses about technology, which can be changed at will according to empirical convenience.

However, as we shall see, these characteristics of the neoclassical models end up generating results that are sometimes considered undesirable in the theories of growth. In fact, the major part of the analytical effort in this area has been about how to reconcile some evident characteristics of the real world within the neoclassical theoretical structure that does not easily accommodate them.

II.2 Solow's model without technical progress

Let us suppose, initially, that there is no technical progress. We will depict this with a Cobb-Douglas 'production function' as follows:

$$Y = F(K, L) = AK^{a}L^{1-a}$$

where 'A' is a constant, 'a' is the share of capital and '1-a' is the share of labour in the product.[3] As a<1 the function incorporates constant returns to scale (a + (1-a) = 1) and decreasing marginal returns for each factor separately (a<1 and (1-a)<1) .

It is important to remember that use of this 'production function' presupposes the logical validity and empirical relevance of the neoclassical model of general equilibrium (Serrano, 2001); that is, it presupposes market clearing in factor markets.

The agents save a constant fraction of the product (produced at full employment and full capacity utilization). Therefore the saving will be:

[3] The peculiarity of the Cobb-Douglas production function is that, due to the (very strong) hypothesis of unitary substitution elasticity, the contribution of each factor to the product is constant; that is, we can take 'a' and '1-a' as parameters. Any other production function, with constant returns to scale, would generate results qualitatively similar. Since our objective is critical, we will always use the simplest formulation.

$$S = sY^*$$

where 's' is the fraction of income saved or the savings ratio.

In terms of the production function, we can see that the growth rate of the economy will be an average of the growth rate of capital and of labour, weighted by the share of each of the factors in the total output. This share in the total product is given by the exponents 'a' and '1-a' of the production function. We can formally define the growth rate of the product in the production function as:

$$g = a\ g_k + (1\text{-}a)\ n$$

where 'g' is the growth rate of the product, 'n' is the growth rate (supposedly exogenous) of the labour force, and 'g_k' is the growth rate of the capital stock.

The growth rate of the capital stock is as follows:

$$g_k = \frac{I}{K}$$

Since, according to neoclassical theory, the potential or full employment [see Serrano, 2001]) saving determines investment, we can replace the level of investment by an expression that defines the level of saving of the economy. Thus we have:

$$g_k = \frac{sY}{K}$$

Y/K = 1/v, where v is the capital-output ratio. Thus, the rate of growth of the capital stock is as follows:

$$g_k = \frac{s}{v}$$

Therefore, the growth rate of the capital stock depends on the saving rate 's'[4] and on how much (potential) output is generated by each unit of existing capital in the economy.

The growth rate of the economy can thus be expressed as follows :

$$g = a\ s/v + (1\text{-}a)\ n$$

[4] Notice that 's' denotes the marginal and average propensity to save. In this paper we will take 's' as given exogenously and will not discuss its determinants. As pointed out by several authors (Solow, 1992; Cesaratto, 1999a, 1999b; Salvadori & Kurz, 1997a, 1997b; Mankiw, 1995), the form chosen to determine 's' does not make much of a difference to the questions treated here. In fact, what happens when we allow, for instance, the savings rate to be determined, a la Ramsey, by a process of dynamic optimization by consumers is that the savings rate becomes elastic in relation to the interest rate. The stronger the consumer's preference for present consumption over future consumption, the higher, *ceteris paribus*, the savings rate will be.

However, every time the labour force, as also the full employment output, grows more quickly than the capital stock (that is, 'n' and therefore 'g' as well are greater than s/v), it will be necessary to keep the labour market in equilibrium; and in order to ensure that all the additional workers are employed, real wages have to fall enough for firms to be encouraged to adopt techniques that are sufficiently labour-intensive and that save capital (the factor that is becoming relatively scarcer). This adoption of less capital-intensive techniques leads to decreases in the capital-output ratio (because in the production function the same K now generates more Y, since the system has incorporated more workers L) . But if 'v' decreases the difference between 'n' and 's/v' falls, since 's/v' increases.

Symmetrically, when the labour force and full employment output grow less rapidly than the capital stock ($n < s/v$ and $g < s/v$), a relative excess of capital occurs. The increase in potential saving will transform itself fully into investment, keeping the equilibrium in the market for capital, only if there is a fall in the rate of interest such that it encourages the adoption of more capital-intensive techniques and techniques that save labour (which become relatively scarcer). This change in capital intensity increases the capital-output ratio (because now the same L generates more Y since it incorporates more capital K). The increase in 'v' reduces 's/v' and progressively reduces the difference between 'n' , 'g' and 's/v' .

Thus, given the endogeneity of 'v' (a result of the neoclassical theory of distribution, based on the equilibrium in factors markets obtained from the operation of the principle of substitution and of the flexibility in prices of these factors), the economy always tends towards a steady state growth path where:[5]

$$v = s/n$$

The equilibrium value of 'v' (which makes $s/v = n$) ensures that the economy grows at the rate:

[5] Notice that, in general, each equilibrium described by the production function is already a long-run equilibrium. A steady-state equilibrium trajectory of growth is a secular sequence of long-run equilibrium positions, which has the property that each new equilibrium position is similar to the previous one except for a scale factor (in our case, the growth rate 'n'). For a discussion of the confusion between long-run equilibrium and trajectories of steady state growth as one of the consequences of the change in method in neoclassical theory with the introduction of the inter-temporal equilibrium idea by Hicks, see Garegnani (1976, 1989) and Serrano (1988, Ch. 2).

$$g = (1-a)n + a\,n$$

or

$$g = n$$

The economy tends towards a path of steady growth in which the growth rate of the product is equal to 'n'. That implies that the steady state growth rate is independent of the savings rate 's' . This occurs because any increase in 's', while it will translate into initial increases of the growth rate of full employment output, will inevitably render the rate of growth of the capital stock higher than the rate of growth of the labour force. Decreasing returns to capital will manifest precisely because, in order to use these increments of the capital stock, it will be necessary, over time, to move towards techniques that use relatively more capital and less labour, which will end up progressively increasing the capital-output ratio and slowing down the rate of growth of the capital stock. This process will continue until the rate of growth of capital, even with the new higher rate of savings, goes back to become equal to the growth rate of full employment output, which, if 'n' does not change, will only occur when the value of 'v' is reduced proportionally to the increase in 's'.

Thus, although the increase of the saving rate permanently affects the level of output, due to decreasing returns to capital accumulation, we have a situation in which 's' does not permanently affect the growth rate of output.

This result was considered undesirable by several authors who believe that there is a strong empirical correlation between the growth rate of the product (and also of the product per worker) and the investment rate, a correlation that is considered to be an important stylized fact in the empirical analysis of long-run growth trends. This dissatisfaction had an important role to play in the development of neoclassical models of endogenous growth (see Solow, 1992; Cesaratto, 1999a, 1999b).

In any case, what is essential to keep in mind is that this independence of the steady state rate of growth relative to 's' depends fundamentally on the existence of decreasing marginal returns to capital and, as we shall see, it happens in any neoclassical growth model that retains this property.

II.3 Solow's model with exogenous technical progress

Technical progress in Solow's original model is of the disembodied kind, in the sense that it does not depend on the introduction of new capital goods and affects equally old machines and new ones. Additionally, it is the kind of disembodied technical progress that is called *labour-augmenting*, whose only effect in the economy is to make each unit of labour become more productive (but not affecting the efficiency of capital). When this kind of technical progress occurs it is as if the economy is combining the same capital stock with a larger quantity of labour, and it makes the full employment level of output increase without an increase in either the labour force or the capital stock.

With technical progress of this kind, the production function is modified as follows:
$Y = AK^a (LH)^{1-a}$

Technical progress in Solow's model, being labour-augmenting, occurs by increases in H while A stays constant. If technical progress is manifested as an increase in A, it would be impossible to keep the economy on a steady-state trajectory, even with a constant rate of saving. This is because technical progress of this kind would necessarily allow, period after period, the same level of installed K to generate more output Y (because A is growing), making the capital/output ratio fall continuously. But if this ratio falls continuously over time and the savings rate is constant, the growth rate of the capital stock of the economy would be accelerating all the time.

It is in order to avoid this that it is always assumed in the neoclassical models that technical progress is labour-augmenting.[6] Otherwise, the model will never tend to a steady state.

We will assume that technical progress grows over time at a rate 'h' which is constant and given **exogenously**. Thus:

$$\frac{\Delta H}{H} = h$$

In this case, the growth rate of the economy depends on one more element: the growth rate of the efficiency of labour. The growth rate of the economy is now given by:

$g = a\ s/v + (1-a)\ (n+h)$

The economy will tend to a steady state just as before, through adjustments in 'v'. The

[6] In non-neoclassical models for equivalent reasons it is supposed that technical progress is Harrod-neutral, but that is another story.

difference is that now the steady-state growth rate is given by the sum of the growth rate of the labour force and of technical progress.

What happens now is that when, say, due to an increase in 's', the growth rate of the capital stock s/v is larger than the sum of the growth rate of the labour force 'n' and the increase of the efficiency of labour 'h', and therefore also larger than the growth of full employment output 'g', it will be possible to absorb this relative excess of capital only if techniques are adopted that economize on labour, techniques that lead to increases in 'v' and later reductions in the growth rate of capital, etc., exactly as we saw above. The only difference here is that technical progress retards the operation of decreasing marginal returns to capital, because it allows capital to grow up to 'h' per cent more than labour, without needing to change the capital-output ratio to reduce the rate of growth of capital. In this way, the steady-state rate of growth of the economy increases by the value of 'h'.[7] Naturally, in the model with technical progress, the economy will be characterized by a rate of growth in the product per worker, in the steady state, which is exactly equal to the growth rate of the efficiency of labour 'h'. In the steady state, the capital-output ratio will be:

$$v = s / (n+h)$$

the growth rate of the economy will be:

$$g = n + h$$

and the growth of product per employed worker will be:

$$g - n = h$$

III. Increasing Returns to Scale in Neoclassical Theories of Exogenous Growth

III.1 Externalities and increasing returns to scale

As is well known, technologies that are characterized by intra-firm increasing returns to scale are not compatible with the neoclassical model of perfect competition. The tendency for costs to decrease would encourage the first firm expand output and dominate the market completely, thus transforming it into a monopoly (see Sraffa,1926, 1930). Therefore, the way in which increasing returns to scale can be accommodated in a

[7] In graphical terms there is a shift of the demand curve for labour and in the demand for capital (but their slopes do not change).

model such as that of Solow's is through certain externalities, which are not taken into account by individual firms which perceive their technology as being characterized by constant returns to scale.[8]

A simple way of representing this in our scheme would be to rewrite the production function as :

$$Y = AK^{a1}(LH)^{a2}$$

with $(a_1+a_2)>1$, $a_1<1$, $a_2<1$.

That is, due to some positive externality for the economy as a whole, now a_1+a_2 is greater than 1, so that the production function is characterized by increasing returns to scale.

We will suppose, however, that both a_1 and a_2 are less than 1, so that decreasing marginal returns to each factor taken separately will still prevail.

In this case it is easy to see that the growth rate of the economy will be given by:

$$g = a_1\, s/v + a_2\, (n + h).$$

However, as the accumulation of capital leads to decreasing returns (a_2 <1) we know that, as before, s/v would deliver a rate of growth which is the same as that of the economy, g, which will in turn depend on the behaviour of exogenous technical progress, of the growth of labour force and of the size of externality that causes increasing returns. Note that h now represents only the growth rate associated with exogenous technical progress which, as we shall see, is no longer equal to the growth of the product per worker, if there are increasing returns to scale. The steady-state rate of growth should now be calculated as:

$$g = a_1 g + a_2\, (n+h)$$

which gives us

$$g = \frac{a_2}{1 - a_1}(n + h)$$

Notice that, if there are increasing returns to scale, the steady-state rate of growth

[8] The other alternative is to suppose imperfect competition in the goods markets. Many modern theories of endogenous growth adopt this path to model the behavior of agents so as to internalise these increasing returns. In this work we will not discuss these versions because our interest is in the hypotheses regarding the "production functions" of the economy, which are fundamentally the same both in the competitive models and in the imperfectly competitive ones (the latter being much more complicated and involving even more ad-hoc hypotheses).

will be higher than the one in the model with constant returns to scale, with the discrepancy depending on the size of the externality (however, it will never be twice as large because, since we assume that each factor is still characterized by decreasing marginal returns, the sum a_1+a_2 will always be smaller than 2). Notice that, in this case, the savings rate still has no effect on the steady-state rate of growth. In this economy, even with the externality that results in increasing returns to scale, capital accumulation still leads to decreasing marginal returns and therefore does not manage to sustain by itself a permanent positive rate of growth without an increase in 'n' or 'h'. Therefore, all that increasing returns to scale can do, given the existence of decreasing marginal returns for capital, is to amplify the effect on the steady-state growth rate of the exogenous increases in 'n' or 'h'.

Moreover there is an additional result. Notice that if the rate of growth of the labour force is increasing, it is not just the rate of growth of output that increases but also the rate of growth rate of the product per worker (because the term $[a_2/(1-a_1)]$ is greater than 1).[9]

The idea that the rate of growth of the product per worker is a positive function of the rate of growth of the endowment of labour is clearly implausible (for, if it were true, we would have observed that countries that have experienced demographic explosions would have grown persistently faster than those that did not). Unfortunately, for those who use the neoclassical notion of market clearing in the markets for factors, this result is inevitable in an economy with increasing returns. Therefore, as has indeed been noted by Solow (Solow (1992)), it is fully possible to incorporate increasing returns to scale in his model. The problem, which Solow curiously does not emphasize, is that this incorporation leads to an absolutely unrealistic result that is not desirable.

III.2 Economies of learning

We can illustrate better the same idea and the same results in another form. Now, instead of talking about a generic externality that produces increasing returns to scale, we are going to refer to an externality generated by capital accumulation through economies

[9] In fact, we can verify that the equilibrium rate of growth of the product per worker is
$g-n = [\,(a_2/(1-a_1)) - 1\,]\,n + [a_2/(1-a_1)]\,h$.

of learning ("learning by doing" in Arrow's sense).

This learning can be formalized in a simple way by making the efficiency of labour H a function of a proxy of the previous experience of the economy. The most obvious candidate to represent this experience is the stock of accumulated capital. Thus:

$H=K^x$

where x measures the effect of the capital stock (via learning) on the efficiency of labour. If we suppose that $x<1$ we obtain decreasing marginal returns for learning by itself.

If we introduce this learning function in the Solow growth model, instead of assuming exogenous technical progress, we have:

$Y=AK^a (LH)^{1-a}$

$Y=AK^a[L(K)^x]^{(1-a)}$

That gives the following expression for the growth rate of the economy:

$g = a\, s/v + (1-a)\, n + [\,(1-a)\, x\,]\, s/v$

where, if we assume that learning is characterized by decreasing returns, the expression between the brackets becomes less than one and therefore capital accumulation, even taking account of the positive learning externality, still has decreasing returns.

On the other hand, the economy, as $x>0$, necessarily has increasing returns to scale equal to $a+(1-a)x+(1-a)$, or $[1+(1-a)x]$, which is clearly less than 2 as in the version presented earlier.

Therefore, we see that the model with learning by doing is virtually identical to the increasing returns model discussed above (apart from the fact that here we ignore the exogenous component of technical progress, which could evidently have been introduced).

Thus, because of the role played by decreasing returns to capital accumulation, the capital stock in this economy ends up growing at the same rate as the product and in the steady-state we have:

$$g = [\, a + (1-a)x\,]\, g + (1-a)\, n$$

$$g = \frac{(1-a)}{1-[a+(1-a)\,x]}\, n$$

which, dividing by (1-a) the numerator and the denominator of the term that multiplies n, becomes

$g= [\ 1/(1-x)\ ]\ n.$

Once again the presence of learning, while retarding the decreasing returns to capital accumulation, does not eliminate it. Therefore, the steady-state rate of growth is still independent of the savings ratio. Moreover, we find here as well, because of increasing returns to scale, the embarrassing result that a higher rate of growth of the labour force leads to an increase in the rate of growth of the product per worker (since $0<x<1$ and therefore $x/(1-x) >0$).[10]

III.3 Embodied technical progress

Nothing prevents us from incorporating in the Solow model the idea that technical progress that increases the workers' efficiency is in good part embodied in new generations of machines, instead of "falling from heaven" in disembodied form. That would lead us to a vintage model, where the most recent "vintages" of capital goods would make the workers equipped with them more efficient. We will not elaborate on this case here so as to avoid mathematical complications. For our purposes it is enough to note that the effect of embodied technical progress acts exactly as that of an externality linked to capital accumulation does. This naturally makes the model exhibit increasing returns to scale. On the other hand, this externality, as any other, it must be assumed, is not strong enough to neutralise the decreasing returns to capital taken separately, or else the economy would lose the mechanism that makes it always tend towards a trajectory of steady-state growth.

But, if in the end the decreasing returns to capital accumulation are not entirely neutralised, we return to a model that produces exactly the same results as that of the Solow model with increasing returns to scale.

Only to illustrate this point, without putting down any further equations, we will resort to an extreme example and assume that the economy we are dealing with only uses circulating capital and therefore the whole capital stock is renewed in each period. In this

[10] Because, in this case, the growth rate of the product per worker is $g-n = [x/(1-x)]\ n$.

case, we would necessarily have to allow for a parameter such as t'x' above that would now represent the elasticity of accumulation with respect to technical progress and be strictly lower than one, so as to ensure that the economy would tend to a steady-state typical of economies with increasing returns. In the case which we are dealing with, this requirement would be represented by equations similar to those in section III.2 and therefore generate the same undesired results. Of course a true vintage model is more complicated than that since, with fixed capital, only part of the capital stock is renewed in each period, but the general principle will necessarily be the same

To conclude, the discussion in this section of the paper shows that, while the neoclassical theory is perfectly capable of accommodating externalities, learning and embodied technical progress (provided that they are labour augmenting), their presence by no means render the theory capable of explaining the association between capital accumulation and growth. Additionally, the introduction of these elements delivers the embarrassing result that an increase in the rate of growth of the labour force increases the rate of growth of per capita product. Thus, not only does further capital accumulation not lead to a steady-state with a higher rate of growth (exactly as in Solow's original model), but also simply accelerating the growth of the labour force automatically solves the problem of economic development!

IV. Neoclassical theories of endogenous growth

IV.1 Constant marginal returns to accumulation

The striking characteristic of the neoclassical theories of endogenous growth is not the endogeneity of technical progress itself, because technical progress is endogenous even in the models discussed in section III above, which are classified as variants of the theory of exogenous growth. What defines these theories as being endogenous in character is that decisions of economic agents (or of the government) with regard to accumulation (saving), which in the neoclassical vision reflects their choice between present and future consumption (of both goods or factors), affect directly the steady state rate of growth of the economy.

We know that this does not occur in Solow's model due to decreasing marginal returns to capital. Thus, the theories of endogenous growth are those in which accumulation is associated with constant and not decreasing marginal returns. Therefore, a larger accumulation effort (in a broad sense) will have the permanent effect of generating a higher steady-state rate of growth. Within the class of endogenous growth models, individual models can be distinguished and grouped together based on the factor that can be accumulated for which constant marginal returns are postulated. For a first group of theories, the factor concerned is physical capital (the so-called AK models). A second group (referred to sometimes as models of the Lucas kind) postulates that it is the accumulation of knowledge (in the form of learning, human capital or even number of designs) which is characterized by constant marginal returns. Thus, the theories of endogenous growth can be classified in two groups: models of constant marginal returns to capital and models of constant marginal returns to knowledge**.**

IV.1 Constant Returns to capital accumulation

a) AK model without growth of the labour force

The central idea underlying this model can be illustrated in simple form using the learning by doing model presented in the previous section.

In that model, the accumulation of capital led to a learning externality that increased workers' efficiency. The externality was measured as:

$H=K^x$, where $x<1$.

The idea underlying the learning by doing model is that this externality, by making capital accumulation automatically generate labour augmenting technical progress, partially compensates the tendency of decreasing returns to capital. That occurs because, when contributing to an increase in the efficiency of labour, this externality has an effect similar to a situation where growth of the stock of capital, always and automatically, increases the rate of growth of the labour force by x percent of the increase in the capital stock. Now, in the so-called AK model, we are assuming that the externality entirely compensates this tendency (because, with x=1, the increase of the capital stock does increase the efficiency of labour by exactly the amount that is necessary to avoid a change in the capital-output ratio). Thus we suppose directly that x=1 and therefore

$H=K$

Replacing this expression in the production function we have

$Y=AK^a(KL)^{(1-a)}$

$Y=AKL^{(1-a)}$

Assuming for now that the labour force does not grow (n=0) and since the product grows proportionally to accumulated capital, the rate of growth of the economy would amount to:

$g = a\, s/v + (1-a)\, s/v$

or

$g = s/v$

and as the labour force is not growing (n=0) the growth of the per capita product (g-n) is also equal to s/v.

As capital accumulation is not characterized by decreasing returns, there is no endogenous tendency to change the capital-output ratio of the economy, contrary to what happens in the Solow model and its variants. Thus, if we double the saving ratio the steady-state growth of capital and of output (both in absolute and per capita terms) doubles permanently.

b) AK model with growth of the labour force

Unfortunately the results are drastically modified if the rate of growth of the labour force is positive (n>0). For in this case the rate of growth in the above model is necessarily given by:

$g = s/v + (1-a)n$

and the growth rate of the product per worker by:

$g - n = s/v + (1-a)\, n - n$

$g - n = s/v – a\, n$

Evidently the model above is incompatible with constant steady-state growth rates. It is clear that if n>0 in each period the product always grows faster than the capital stock. Therefore, period after period, the parameter v (the capital-output ratio) will be decreasing and therefore the growth rate of the capital stock will be increasing relative to the previous period. But to this higher rate of growth of the stock of capital corresponds an even higher growth rate of the product in the following period, and so on. Any positive n will make the rate of growth corresponding to a given saving ratio 's' accelerate continually.The same will be true of the rate of growth of per capita productbecause with the parameter 'v' decreasing continually for a given 's' the per capita growth rate increases without limit. Therefore if the labour force is growing, rather than accumulate capital more quickly to seize the externality, what agents should do is to save very little and generate a demographic explosion, which does not need to be too big because any positive rate of growth of the population quickly leads the economy to growth rates that tend to infinity!

The result is even more implausible than that of the learning by doing model in which, due to increasing returns in the economy, a constant rate of growth of the labour force generated a positive per capita growth rate. The reason for this even less reasonable result is that the learning by doing model still retained decreasing returns to capital, which guaranteed that a constant positive growth rate of the population did not manage to continually accelerate the growth rate, since there was a counteracting tendency in the form of an increase in the capital-output ratio. This tendency is neutralized by assumption in the AK model with $n>0$ and therefore the growth rate rises without limit.[11]

c) AK model with the Frankel modifier

One way to avoid the above result is through a different specification of the externality that is known as the Frankel modifier.[12]

In this case, we introduce the hypothesis that the positive externality from capital accumulation is not a function of the absolute stock of accumulated capital, but of the capital stock accumulated per worker.

Thus:

$$H = (K/L)^x$$

Assuming additionally that the externality is such that it exactly neutralises the tendency to decreasing returns arising from the increase in the K/L, we have $x=1$

and therefore:

$$H = K/L$$

which is the Frankel modifier.

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[11] An intuitive way of understanding what is occurring is to remember that the AK model is nothing more than the learning model with x being equal to 1. In the equation that shows the growth rate of that model we see that if $x=1$ the denominator falls to zero and the growth rate $g=n/(1-x)$ tends to infinity.

[12] Frankel's Model was "rediscovered" by Cesaratto (1995, 1999a and 1999b) as the endogenous growth model that shows more clearly the meaning and limitations of this approach.

Using this modifier and replacing it in the production function, we have:

$Y = AK^{a}[(K/L)\ L]^{(1-a)}$

$Y = AK$

The equation above shows that only in this case is it guaranteed that the growth in the product is in fact proportional to that of capital stock (which is what is implied by the title AK model), even if n is positive.

In this case, even with $n>0$, the growth rate of the economy is in fact given by:

$g = s/v$

while the growth of per capita product is given by:

$g - n = s/v - n$

In this model in fact there is no endogenous tendency for the capital-output ratio to change and the steady state rates of growth, both in absolute and per capita terms, are endogenous because they depend positively on the savings ratio.

However the model ends up with the curious result that, taking into account the externality, the contribution of the growth of the labour force to the growth of the product is zero. This is because the whole of the increase in the growth of the product due to the employment of more workers n(1-a) is compensated by the technical regress induced by the presence of more workers in relation to the capital stock (the denominator of K/L increases in the same proportion to reduce the rate of growth rate by n(1-a)) .

Notice how this result implies that the model is characterized, after all, by **constant** returns to scale because the marginal returns to capital are equal to 1 and that of labour is zero (if labour had any positive contribution we would have increasing returns to scale which, as we saw in item ‘a’ above, would combine with constant marginal returns to capital to deliver a rate of growth equal to infinity). What the modifier does is to cancel labour's contribution to the product since the growth of the labour force has the effect of decreasing the product per worker proportionately (as becomes clear from the equation representing per capita product growth which is a negative function of n). With the modifier, labour has a big negative externality that is not easy to explain, except in term of the technical need to eliminate the contribution of labour.

It is important, in spite of these sufficiently strange results, to stress that, again, even with the modifier, if x is greater than 1 by a very small margin, the model tends to generate explosive rates of growth because then capital accumulation by itself has increasing marginal returns.

On the other hand, if x is smaller than 1 even by a little (say 0.99), the model reverts to something very close to the previous learning by doing model that does not generate endogenous growth. In fact, if we keep the modifier, the model with $x<1$ will tend to a stationary state because the contribution of the labour force to growth is zero and capital has decreasing marginal returns. Thus the AK model with the modifier only works when x is exactly equal to 1 because any small deviation from this leads to zero or infinite rates of growth.

IV.3 Constant Returns to Knowledge

We now turn to the second family of endogenous growth models in which knowledge is the factor that is accumulated under constant marginal returns[13], while for physical capital the usual characteristic of decreasing marginal returns is retained (Lucas type models). These models are also called two sector models because there is a second sector in the economy that produces the increases in the stock of knowledge.

a) Constant returns to knowledge with a constant labour force

We start with the case in which the labour force does not grow. The production function for the sector that produces goods is given by:

$$Y = AK^{a}((1-z)LH)^{(1-a)}$$

where the only novelty is the parameter (1-z), which measures the proportion of the labour force employed in this sector that produces goods. Thus, 'z' is the proportion of the labour force employed in second sector that produces knowledge, which transforms itself directly into increases in the efficiency of labour H. This knowledge production sector uses the following production function:

[13] The idea that "knowledge" can be treated as a "production factor" is not very easy to accept. It requires *at the least* the possibility of defining a plausible theoretical index of quantity of accumulated "knowledge" that is **cardinal**. If in fact "knowledge" is not a "thing" that can be summed and subtracted easily (as of course it doesn't seem to be) it does not make any logical sense, as pointed out by Steedman(2001), to talk about constant or non-constant returns. Also, according to Steedman, until now there is no valid justification for this supposition about cardinality in the literature, which invalidates this whole family of models.

$\Delta H = j\,(z\,L)\,H$

which shows that the product of this sector, the new knowledge (equal to the increase in H), is produced by means of knowledge and labour (z percent of the labour force).

Notice that, as j is supposed to be given, the technology that produces knowledge by means of knowledge has constant marginal returns.

The growth rate of knowledge h is given by :

$\Delta H/H = j\,z\,L$

In this case we see that we are dealing with a model very similar to that of Solow's, with the difference that technical progress instead of being exogenous is explained by the technology for production of knowledge and the fraction of the labour force employed in that sector (z).

We have then that the growth rate of the economy is given by:

$g = a\,(s/v) + (1\text{-}a)\,j\,z\,L$

where, due to the decreasing marginal returns to physical capital accumulation, s/v will tend to g, which in the steady-state will be:

$g= j\,z\,L$

which is both the growth rate of output and of per capita output, in the case in which the labour force does not grow.

Notice that the equation above generates endogenous growth to the extent that society's decision to employ proportionately more people in the knowledge production sector allows a permanent increase in the growth rate of the economy. Therefore, although in this model the accumulation of physical capital continues to be subject to decreasing returns, the accumulation of knowledge has constant marginal returns and therefore, given the technology that makes possible the production of knowledge by knowledge with constant returns, a higher rate of accumulation of knowledge will lead to a higher growth rate of the product.

Notice that even taking this simple case this kind of model, contrary to the AK model, cannot explain any stylized relation between the share of investment in income and the rate of growth of the product (both in absolute and per worker terms). This is because the characteristic of decreasing marginal returns to physical capital is being retained. Here, the part of accumulation that does not generate decreasing marginal returns is that of knowledge, and the relevant "savings" ratio is the proportion of the labour force allocated to the sector that produces the increase in the stock of knowledge (the saving is done directly in terms of the primary factor labour). The loss of present consumption comes from the fact that less goods will be produced today if z is increased.

b) Constant returns to knowledge with an increasing labour force

Suppose now that the labour force is growing ($n>0$). The growth rate of the economy is given by:

$$g = a\, s/v + (1-a)\,(j\, z\, L + n)$$

Again, as s/v always tends to g, we could be led to think that we would have in the steady-state:

$$g = j\, z\, L + n$$

and product per worker :

$$g - n = j\, z\, L$$

However, this evidently will **not** occur since, given the effect of the size of the labour force L in the equations for the growth rates, any positive n will accelerate the (absolute and per capita) growth rate. This occurs because in each period, for the same values of j and z, we will necessarily have a larger L. The model is not able to generate a stable rate of growth because the **growth rate** of the efficiency of labour is a function of the **absolute** number of people employed in the knowledge production sector. If we keep constant the ratio z of the number employed in this sector to the labour force, the absolute number of workers in this sector will expand if the labour force grows and, with it, the growth rate of the economy would expand without limit.[14]

c) The Lucas modifier

Using the knowledge production technology described above, it is impossible for a model with constant returns to knowledge (represented by a given and constant

[14] The problem with this class of neoclassical models of endogenous growth, including Romer (1990), Grossman-Helpman (1989) and Aghion-Howitt (1992), was originally pointed out by one of us, (Cesaratto, 1995, p.25), and by Jones (1995), which called it the “scale effect” (see also Michl, 2000). Cesaratto (1995) also showed this effect in models of Phelps (1966) and Shell (1966).

parameter j) to generate steady state growth if the labour force grows.

The "solution" to this problem was obtained by Lucas through the artifact of changing the specification of the technology in the sector that produces knowledge, which is transformed into:

$\Delta E = j\ z\ E$

where E is defined as units of labour, already measured in efficiency units, i.e.,

$E = L\ H$

Thus this technology produces variations in the quantity of labour, measured in efficiency units. This means that the function not only deals with the variations in H, but also embodies the variations in L, since the two ways to increase the units of labour, measured in efficiency units, are increasing the efficiency of each worker and increasing the number of workers.

We canl rewrite the equation above making this fact explicit as:

$\Delta\ (LH) = j\ z\ LH$

thus

$\Delta\ (LH) / (LH) = j\ z$

We thus see that the change in labour measured in efficiency units can be decomposed in its two components:

$\Delta\ (LH) / (LH) = n + h$

which allows us then to write the rate of growth of the efficiency of labour explicitly as:

$n + h = j\ z$

$h = j\ z - n$

The only justification for the above equation would be that the increase in efficiency does not depend on knowledge accumulation by itself, but on knowledge accumulation per worker (in a strict analogy with the Frankel modifier that made efficiency increase with the quantity of physical capital per worker).

Once we have this result it is easy to calculate the rate of growth of the economy as:

$g = a\ s/v + (1-a)\ (j\ z - n + n)$

$g = j\ z - n + n$

$g = j\ z$

while the per capita growth rate is given by:

$g - n = j\ z - n$

We see that a "modifier", such as the one by Frankel, introduces the hypothesis that increases in the labour force do not increase the product, because their usual positive effect is fully compensated by a negative externality, that makes the efficiency of labour fall proportionately.

In the end, the Lucas type growth models, as well as the AK models, have problems when n is positive that can only be solved if we eliminate the contribution of labour to the product. Although the "solution" is very similar, the problem, in the case of the Lucas type of model when n is positive, is very different from the problem in the AK model. In the case of the model with constant returns to knowledge, the problem is that in the sector where knowledge is produced, the growth rate of the knowledge 'factor' is a positive function of the absolute size of the labour force and tends to accelerate without end for any positive n.

Curiously, in this kind of model, it is the labour force that generates a strong positive externality, since an increase in the number of people working in the knowledge production sector automatically increase the rate of growth of labour efficiency. Thus, a device such as the Lucas modifier is essential to generate a negative externality that entirely cancels this positive externality.

d) Lucas-type model with increasing returns to scale.

The model with constant returns to knowledge, in its version with the "modifier", has also the advantage of being compatible with the existence of increasing returns caused by some externality derived from the accumulation of physical capital, without generating the implausible result that the steady state growth rate is a function of the growth rate of the labour force. It is important to notice that this externality cannot be strong enough to cancel entirely the decreasing marginal returns to capital.

Let us suppose, keeping the knowledge production function with the Lucas modifier that the following production function is used in the goods sector:

$Y = A\ K^{a}\ (1\text{-}z)\ (L\ H)^{\ (1\text{-}a)}\ B$

where B is an element that measures the externality derived from capital accumulation, say because of learning by doing effects. We have then:

$B = K^b$

where the parameter 'b' measures the contribution of the externality to the growth of the product.

The growth rate of the product is given by:

$g = a\ s/v + (1-a)\ (j.z-n\ +\ n) + b\ s/v$

If a+b<1 we still have in spite of the externality decreasing marginal returns to physical capital. Therefore s/v will tend to g. It follows then that:

$$g = \frac{1-a}{1-(a+b)}(jz)$$

and the product per capita product grows at the rate :

$$g - n = \frac{1-a}{1-(a+b)}(jz) - n$$

We see that the growth rate, even with increasing returns to scale, does not depend at all on the growth of the labour force. On the contrary, the growth rate of the product per worker will depend negatively (and not positively) on the growth of the labour force. This is ensured only because the Lucas modifier conveniently eliminates the positive contribution of labour to the product and the existence of the sector that produces knowledge by means of knowledge, with constant marginal returns, keeps the rate of growth positive. In this specific case (which is quite close to the original analysis by Lucas), increasing returns to scale only amplify the growth rate of the economy, which is sustained by the accumulation of knowledge.

Finally, it is important to remember that even with these extreme and arbitrary hypotheses about technical progress, none of these versions of the Lucas type of model manage to explain the stylized facts that relate the accumulation of physical capital to the growth of the product and of the product per worker.

V. Three final remarks

V.1 Capital accumulation in a classical framework

The truth is that it is not easy to explain the stylized facts relating capital accumulation and growth, in a coherent form, with the premises of the neoclassical theory of distribution. It is very easy to illustrate this point if, in the context of the simple analytical scheme that we are using in this paper, we now abandon the marginalist explanation of distribution and suppose that the real wage is given exogenously by the economic and social forces described by the economists of the classical surplus approach.

In this case, we abandon the equilibrium condition that labour demand has to adapt itself to an exogenous endowment of labour and assume, like the classics (see Serrano (2001)), that under capitalism it is the growth of the labour force that secularly follows the growth of employment opportunities. Thus:

$n = g - h$

where technical change now is seen as Harrod-neutral, that is, it does not change significantly the capital-output ratio acting only on the labour coefficient.

In this scheme, given the real wage, the firms choose the most profitable technique among those available thereby determining the capital-output ratio of the economy. On the other hand, this level of the real wage, together with the level of the product per worker of the chosen technique, will determine the rate of profit of the system, which, given the proportion of profits that is consumed, will determine the savings ratio 's'. Assuming, temporarily, that Say's Law is valid, we can take it that the investment share is determined directly by this savings ratio.

Since there is no scarcity of labour, producers increase output at the rate:

$g = s / v$

period after period, without capital accumulation leading to any kind of decreasing marginal returns.

Therefore, we see that merely replacing the neoclassical explanation of distribution by a classical one, allows us to immediately and easily explain the positive relation between the share of investment and the rate of growth of the product.

Moreover, if we make the additional assumption that the Harrod-neutral technical progress is really endogenous, in the sense that its rhythm depends on the growth rate of the economy, we easily obtain a positive relation between the growth of the product per worker and the investment share.

In a simple way, we can introduce the technical progress function as:

$h = h_1 + h_2 g$

obtaining :

$h = h_1 + h_2 \, s/v$

and therefore

$g - n = h_1 + h_2 \, s/v$

Thus, the stylized facts, which seemed so difficult to explain with the neoclassical models, are easily explained in a scheme based on the classical approach, purely because that scheme is free from the theoretical straight-jacket created by the notion that the labour force is "scarce" under capitalism. This route was taken by Kurz and Salvadori in several contributions (e.g. 1997a, 1997b).[15]

Of course, in the above discussion the unsatisfactory element was the appeal to "Say's law". However, the classical scheme in no way requires that hypothesis. It is perfectly possible to think of a classical system in which the growth rate of the economy is determined by the evolution of effective demand, in particular by the rate of growth of the autonomous components of final demand. From there, through a flexible accelerator mechanism, the share of investment adjusts itself to the rate of growth of demand. On the other hand, the multiplier effect makes the saving ratio adjust to the investment share. In a scheme of this type, known as classical supermultiplier (which will not be developed here, see Serrano (1996)), we can entirely dispense with Say's law and keep the classical explanation, delineated above, of the positive relation between capital accumulation and the growth of the product and productivity. An analysis of the relationship between technological change and long period effective demand, which adopts the supermultiplier approach, is found in Cesaratto, Stirati and Serrano (2003).

V.2 The practical need for the modifier

After a wave of initial enthusiasm, neoclassical theories of endogenous growth, due to their strict dependence on exact and extreme values of the parameters, has in general lost terrain to variants of the Solow model with externalities, which supposedly explain better the stylized facts of the economic development (see particularly

[15] Notice that Kurz & Salvadori (1997a, 1997b) overestimate the similarities between the neoclassical theory of endogenous growth and the classical approach because they do not seem to take into account, as we saw above, that the neoclassical models of endogenous growth retain the full employment of the labour force condition.

Mankiw(1995)).

However, the authors who followed this route do not seem to have taken sufficiently into account the peculiar corollary that this type of model has, in which the rate of growth of output per worker is a positive function of the rate of growth of the labour force.[16]

We saw above that in the Lucas type of models there is a way of introducing increasing returns to scale through externalities that avoids such a result. The introduction of this externality in the Lucas model eliminates the undesirable effect of the growth of the labour force, through the totally arbitrary expedient of a “modifier”. Notices that Lucas' arbitrary assumption has an consequence of economic significance, in that it shows that neoclassical models of endogenous growth manage to consider the impact of the structure of the labour force (or of R&D) on technological change only by excluding any role for the scale of the activity. But that would mean that Luxembourg could obtain the same rate of technological progress as the USA, so long as it has the same share of the labour force employed in R&D (Jones, 1995; Cesaratto, 1999b).

We must note that, contrary to what neoclassical authors say, it is the endogenous growth model of Lucas, with all of its arbitrary assumptions and not the less arbitrary Solow model that is being used as the basis for neoclassical studies on the contribution of capital accumulation to economic growth. Therefore, these studies presuppose the empirical validity of the curious (but convenient) specification of the knowledge production technology used by Lucas. It is ironic to note that the Lucas' model was introduced originally in the Marshall lectures series in Cambridge, England. Almost a century after Marshall, neoclassical economists have made no progress whatsoever in resolving the difficulties they face in reconciling the obvious importance of capital accumulation for growth and technical progress with the premises of the theory of distribution based on the idea of a "full employment" real wage.

V.3 A reminder of the Sraffian capital critique

We should just mention here that since the sixties Sraffian critics have been

[16] Mankiw (1995) does not mention this "small" problem.

demonstrating, among other things, that it is not possible to deduce logically the principle of (direct or indirect) substitution between factors in any neoclassical model in which there are heterogeneous capital goods (see Serrano (2001,2002)).

These critiques, which were never refuted, imply that it is practically impossible to extend the results of the neoclassical models, such as the ones discussed above, beyond the context of an economy that produces a single homogeneous capital good.

However, the criticism continues to be ignored, with the argument that aggregate or macroeconomic models are always less rigorous in comparison with models of general equilibrium, but are very useful and absolutely necessary in some applications.

The argument about the usefulness of simple models is correct but irrelevant. In reality, the Sraffian criticism means precisely that the disaggregated and supposedly more rigorous version of the neoclassical theory is exactly the one that does not manage to guarantee the results that it should, that is, to explain distribution coherently based on the operation of the forces of supply and demand.

Therefore is not easy to understand why the immense majority of those who study economic growth continue to use as the base for their simple models an idea (that of labour as a scarce factor) which simultaneously: (1) seems deprived of empirical content; (2) creates the analytical difficulties that we have discussed in this paper and (3) it cannot be as being based on more complex and rigorous versions of neoclassical theory.

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